

Quiz 11

March 1, 2017

Show all work and circle your final answer.

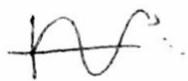
1. (5 points) A region R is bounded above by $y = f(x)$, below by $y = g(x)$, on the left by $x = 4$, and on the right by $x = 10$. Assume constant density ρ . What is M_x (in terms of $f(x)$ and $g(x)$)? What is M_y ?

$$\boxed{\begin{aligned} M_x &= \rho \int_4^{10} \frac{1}{2} (f(x)^2 - g(x)^2) dx \\ M_y &= \rho \int_4^{10} x (f(x) - g(x)) dx \end{aligned}} \quad (\text{by definition})$$

2. (5 points)

- (a) Does the function $y = \sin(2\pi x)$ have a limit as x approaches infinity? If so, what is the limit?

No.



This function is oscillating between -1 and 1, so there is no limit.

- (b) Is the sequence $a_n = \sin(2\pi n)$ ($n \geq 1$) convergent? If so, what does it converge to?

Yes.

$\lim_{n \rightarrow \infty} a_n = 0$

n	1	2	3	...	n	...
a_n	0	0	0		0	

since $\sin(2\pi n) = 0$ for every integer n

3. (5 points) Is the sequence $a_n = \frac{n}{n^2 + 1}$ increasing, decreasing, or neither? Show all work.

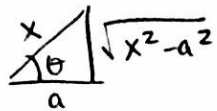
Let $f(x) = \frac{x}{x^2 + 1}$.

$$f'(x) = \frac{x^2 + 1 - x(2x)}{(x^2 + 1)^2} = \frac{-x^2 + 1}{(x^2 + 1)^2} \leq 0 \text{ if } x \geq 1$$

So a_n is decreasing.

Note: we can't only check for $n=1, 2, 3$. Try $a_n = (n-3)^2$, $n \geq 1$, which is neither increasing nor decreasing.

4. (5 points) To integrate $\int \frac{dx}{\sqrt{x^2 - a^2}}$ ($a > 0$), we use the trig substitution $x = a \sec \theta$, substitute, then integrate to get $\ln |\sec \theta + \tan \theta| + C$. Back substitute to find the solution in terms of x .



$$\sec \theta = \frac{x}{a}$$
$$\tan \theta = \frac{\sqrt{x^2 - a^2}}{a}$$

$$\text{So } \ln |\sec \theta + \tan \theta| + C = \boxed{\ln \left| \frac{x}{a} + \sqrt{\frac{x^2 - a^2}{a}} \right| + C}$$